

Risk Analysis of the Space Shuttle: Pre-Challenger Prediction of Failure

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In this document we reperform some of the analysis provided in *Risk Analysis of the Space Shuttle: Pre-Challenger Prediction of Failure* by Siddhartha R. Dalal, Edward B. Fowlkes, Bruce Hoadley published in *Journal of the American Statistical Association*, Vol. 84, No. 408 (Dec., 1989), pp. 945-957 and available at <http://www.jstor.org/stable/2290069>.

On the fourth page of this article, they indicate that the maximum likelihood estimates of the logistic regression using only temperature are: $\hat{\alpha} = \mathbf{5.085}$ and $\hat{\beta} = \mathbf{-0.1156}$ and their asymptotic standard errors are $s_{\hat{\alpha}} = \mathbf{3.052}$ and $s_{\hat{\beta}} = \mathbf{0.047}$. The Goodness of fit indicated for this model was $G^2 = \mathbf{18.086}$ with **21** degrees of freedom. Our goal is to reproduce the computation behind these values and the Figure 4 of this article, possibly in a nicer looking way.

Technical information on the computer on which the analysis is run

We will be using the R language using the ggplot2 library.

```
library(ggplot2)
sessionInfo()

## R version 3.5.1 (2018-07-02)
## Platform: i386-w64-mingw32/i386 (32-bit)
## Running under: Windows 7 (build 7601) Service Pack 1
##
## Matrix products: default
##
## locale:
## [1] LC_COLLATE=French_France.1252 LC_CTYPE=French_France.1252
## [3] LC_MONETARY=French_France.1252 LC_NUMERIC=C
## [5] LC_TIME=French_France.1252
##
## attached base packages:
## [1] stats      graphics  grDevices  utils      datasets  methods   base
##
## other attached packages:
## [1] ggplot2_3.0.0
##
## loaded via a namespace (and not attached):
## [1] Rcpp_0.12.18      rstudioapi_0.7    knitr_1.20        magrittr_1.5
## [5] munsell_0.5.0     colorspace_1.3-2  rlang_0.2.1       stringr_1.3.1
## [9] plyr_1.8.4        tools_3.5.1       grid_3.5.1        gtable_0.2.0
## [13] withr_2.1.2       htmltools_0.3.6   yaml_2.2.0        lazyeval_0.2.1
## [17] rprojroot_1.3-2   digest_0.6.16     tibble_1.4.2      crayon_1.3.4
## [21] evaluate_0.11     rmarkdown_1.8     stringi_1.1.7     compiler_3.5.1
## [25] pillar_1.3.0      scales_0.5.0      backports_1.1.2
```

Here are the available libraries

```
devtools::session_info()
```

```
## Session info -----
##   setting  value
##   version  R version 3.5.1 (2018-07-02)
##   system   i386, mingw32
##   ui       RTerm
##   language (EN)
##   collate   French_France.1252
##   tz       Europe/Paris
##   date     2018-10-10

## Packages -----
##   package      * version date          source
##   backports     1.1.2   2017-12-13 CRAN (R 3.5.0)
##   base          * 3.5.1   2018-07-02 local
##   colorspace    1.3-2   2016-12-14 CRAN (R 3.5.1)
##   compiler      3.5.1   2018-07-02 local
##   crayon        1.3.4   2017-09-16 CRAN (R 3.5.1)
##   datasets      * 3.5.1   2018-07-02 local
##   devtools      1.13.6  2018-06-27 CRAN (R 3.5.1)
##   digest        0.6.16  2018-08-22 CRAN (R 3.5.1)
##   evaluate      0.11    2018-07-17 CRAN (R 3.5.1)
##   ggplot2       * 3.0.0   2018-07-03 CRAN (R 3.5.1)
##   graphics      * 3.5.1   2018-07-02 local
##   grDevices     * 3.5.1   2018-07-02 local
##   grid          3.5.1   2018-07-02 local
##   gtable        0.2.0   2016-02-26 CRAN (R 3.5.1)
##   htmltools     0.3.6   2017-04-28 CRAN (R 3.5.1)
##   knitr         1.20    2018-02-20 CRAN (R 3.5.1)
##   lazyeval      0.2.1   2017-10-29 CRAN (R 3.5.1)
##   magrittr      1.5     2014-11-22 CRAN (R 3.5.1)
##   memoise       1.1.0   2017-04-21 CRAN (R 3.5.1)
##   methods      * 3.5.1   2018-07-02 local
##   munsell       0.5.0   2018-06-12 CRAN (R 3.5.1)
##   pillar        1.3.0   2018-07-14 CRAN (R 3.5.1)
##   plyr          1.8.4   2016-06-08 CRAN (R 3.5.1)
##   Rcpp          0.12.18 2018-07-23 CRAN (R 3.5.1)
##   rlang         0.2.1   2018-05-30 CRAN (R 3.5.1)
##   rmarkdown     1.8     2017-11-17 url
##   rprojroot     1.3-2   2018-01-03 CRAN (R 3.5.1)
##   rstudioapi    0.7     2017-09-07 CRAN (R 3.5.1)
##   scales       0.5.0   2017-08-24 CRAN (R 3.5.1)
##   stats        * 3.5.1   2018-07-02 local
##   stringi      1.1.7   2018-03-12 CRAN (R 3.5.0)
##   stringr      1.3.1   2018-05-10 CRAN (R 3.5.1)
##   tibble       1.4.2   2018-01-22 CRAN (R 3.5.1)
##   tools        3.5.1   2018-07-02 local
##   utils        * 3.5.1   2018-07-02 local
##   withr        2.1.2   2018-03-15 CRAN (R 3.5.1)
##   yaml         2.2.0   2018-07-25 CRAN (R 3.5.1)
```

Loading and inspecting data

Let's start by reading data:

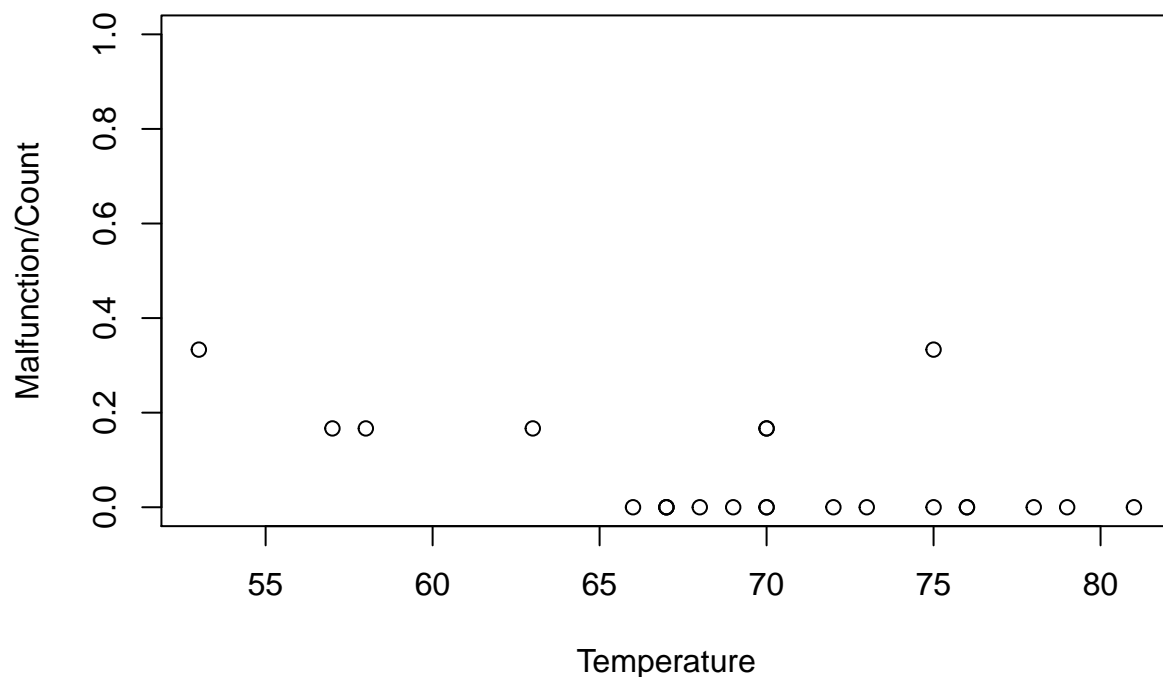
```
data = read.csv("https://app-learninglab.inria.fr/gitlab/moocrr-session1/moocrr-reproducibility-study/r  
data
```

##	Date	Count	Temperature	Pressure	Malfunction
## 1	4/12/81	6	66	50	0
## 2	11/12/81	6	70	50	1
## 3	3/22/82	6	69	50	0
## 4	11/11/82	6	68	50	0
## 5	4/04/83	6	67	50	0
## 6	6/18/82	6	72	50	0
## 7	8/30/83	6	73	100	0
## 8	11/28/83	6	70	100	0
## 9	2/03/84	6	57	200	1
## 10	4/06/84	6	63	200	1
## 11	8/30/84	6	70	200	1
## 12	10/05/84	6	78	200	0
## 13	11/08/84	6	67	200	0
## 14	1/24/85	6	53	200	2
## 15	4/12/85	6	67	200	0
## 16	4/29/85	6	75	200	0
## 17	6/17/85	6	70	200	0
## 18	7/29/85	6	81	200	0
## 19	8/27/85	6	76	200	0
## 20	10/03/85	6	79	200	0
## 21	10/30/85	6	75	200	2
## 22	11/26/85	6	76	200	0
## 23	1/12/86	6	58	200	1

We know from our previous experience on this data set that filtering data is a really bad idea. We will therefore process it as such.

Let's visually inspect how temperature affects malfunction:

```
plot(data=data, Malfunction/Count ~ Temperature, ylim=c(0,1))
```



Logistic regression

Let's assume O-rings independently fail with the same probability which solely depends on temperature. A logistic regression should allow us to estimate the influence of temperature.

```
logistic_reg = glm(data=data, Malfunction/Count ~ Temperature, weights=Count,
                    family=binomial(link='logit'))
summary(logistic_reg)
```

```
##
## Call:
## glm(formula = Malfunction/Count ~ Temperature, family = binomial(link = "logit"),
##      data = data, weights = Count)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.95227  -0.78299  -0.54117  -0.04379   2.65152
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   5.08498    3.05247   1.666  0.0957 .
## Temperature  -0.11560    0.04702  -2.458  0.0140 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
```

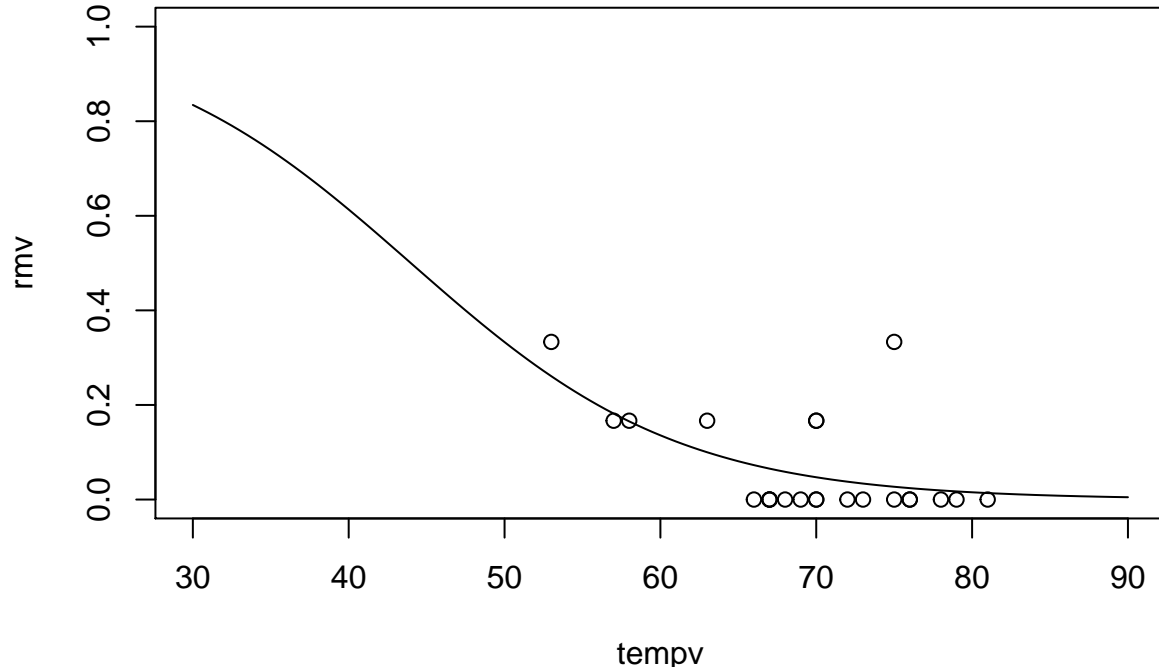
```
##
## Null deviance: 24.230 on 22 degrees of freedom
## Residual deviance: 18.086 on 21 degrees of freedom
## AIC: 35.647
##
## Number of Fisher Scoring iterations: 5
```

The maximum likelihood estimator of the intercept and of Temperature are thus $\hat{\alpha} = \mathbf{5.0850}$ and $\hat{\beta} = \mathbf{-0.1156}$ and their standard errors are $s_{\hat{\alpha}} = \mathbf{3.052}$ and $s_{\hat{\beta}} = \mathbf{0.04702}$. The Residual deviance corresponds to the Goodness of fit $G^2 = \mathbf{18.086}$ with **21** degrees of freedom. **I have therefore managed to replicate the results of the Dalal *et al.* article.**

Predicting failure probability

The temperature when launching the shuttle was 31°F. Let's try to estimate the failure probability for such temperature using our model :

```
# shuttle=shuttle[shuttle$r!=0,]
tempv = seq(from=30, to=90, by = .5)
rmv <- predict(logistic_reg,list(Temperature=tempv),type="response")
plot(tempv,rmv,type="l",ylim=c(0,1))
points(data=data, Malfunction/Count ~ Temperature)
```

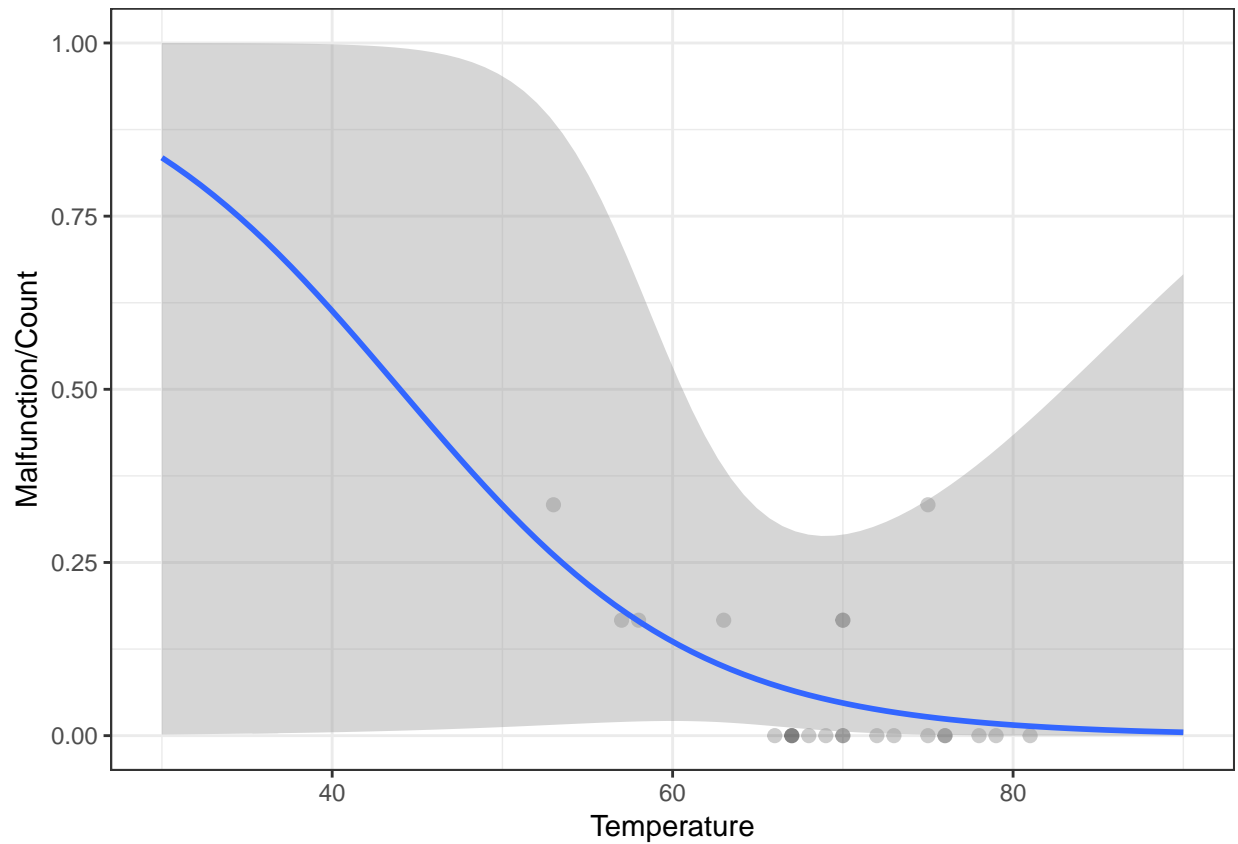


This figure is very similar to the Figure 4 of Dalal *et al.* **I have managed to replicate the Figure 4 of the Dalal *et al.* article.**

Let's try to plot confidence intervals although I am not sure exactly how they are computed.

```
ggplot(data, aes(y=Malfunction/Count, x=Temperature)) + geom_point(alpha=.2, size = 2) +
  geom_smooth(method = "glm", method.args = list(family = "binomial"), fullrange=T) +
  xlim(30,90) + ylim(0,1) + theme_bw()
```

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```



No confidence region was given in the original article. **Let's hope this confidence region estimation is correct.**