# Risk Analysis of the Space Shuttle: Pre-Challenger Prediction of Failure

*Arnaud Legrand*

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In this document we reperform some of the analysis provided in *Risk Analysis of the Space Shuttle: Pre-Challenger Prediction of Failure* by *Siddhartha R. Dalal, Edward B. Fowlkes, Bruce Hoadley* published in *Journal of the American Statistical Association*, Vol. 84, No. 408 (Dec., 1989), pp. 945-957 and available at http://www.jstor.org/stable/2290069.

On the fourth page of this article, they indicate that the maximum likelihood estimates of the logistic regression using only temperature are: $\hat{\alpha} = 5.085$ and $\hat{\beta} = -0.1156$ and their asymptotic standard errors are $s_{\hat{\alpha}} = 3.052$ and $s_{\hat{\beta}} = 0.047$. The Goodness of fit indicated for this model was $G^2 = 18.086$ with 21 degrees of freedom. Our goal is to reproduce the computation behind these values and the Figure 4 of this article, possibly in a nicer looking way.

## Technical information on the computer on which the analysis is run

We will be using the R language using the ggplot2 library.

```r
library(ggplot2)
sessionInfo()
```

```
## R version 3.5.1 (2018-07-02)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Debian GNU/Linux buster/sid
##
## Matrix products: default
## BLAS: /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.8.0
## LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.8.0
##
## locale:
##  [1] LC_CTYPE=fr_FR.UTF-8       LC_NUMERIC=C
##  [3] LC_TIME=fr_FR.UTF-8        LC_COLLATE=fr_FR.UTF-8
##  [5] LC_MONETARY=fr_FR.UTF-8    LC_MESSAGES=fr_FR.UTF-8
##  [7] LC_PAPER=fr_FR.UTF-8       LC_NAME=C
##  [9] LC_ADDRESS=C               LC_TELEPHONE=C
## [11] LC_MEASUREMENT=fr_FR.UTF-8 LC_IDENTIFICATION=C
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] ggplot2_3.0.0
##
## loaded via a namespace (and not attached):
##  [1] Rcpp_0.12.17     bindr_0.1.1      knitr_1.20       magrittr_1.5
##  [5] tidyselect_0.2.4 munsell_0.5.0    colorspace_1.3-2 R6_2.2.2
##  [9] rlang_0.2.1      stringr_1.3.1    plyr_1.8.4       dplyr_0.7.6
## [13] tools_3.5.1      grid_3.5.1       gtable_0.2.0     withr_2.1.2
```

```
## [17] htmltools_0.3.6  yaml_2.2.0       lazyeval_0.2.1   rprojroot_1.3-2
## [21] digest_0.6.15    assertthat_0.2.0 tibble_1.4.2     bindrcpp_0.2.2
## [25] purrr_0.2.5      glue_1.2.0       evaluate_0.10.1  rmarkdown_1.10
## [29] stringi_1.2.3    compiler_3.5.1   pillar_1.2.3     scales_0.5.0
## [33] backports_1.1.2  pkgconfig_2.0.1
```

Here are the available libraries

```r
devtools::session_info()
```

```
## Session info -------------------------------------------------------------

##  setting  value
##  version  R version 3.5.1 (2018-07-02)
##  system   x86_64, linux-gnu
##  ui       X11
##  language (EN)
##  collate  fr_FR.UTF-8
##  tz       Europe/Paris
##  date     2018-09-23

## Packages -----------------------------------------------------------------

##  package    * version date       source
##  assertthat   0.2.0   2017-04-11 CRAN (R 3.5.0)
##  backports    1.1.2   2017-12-13 CRAN (R 3.5.1)
##  base       * 3.5.1   2018-07-02 local
##  bindr        0.1.1   2018-03-13 CRAN (R 3.5.0)
##  bindrcpp     0.2.2   2018-03-29 CRAN (R 3.5.0)
##  colorspace   1.3-2   2016-12-14 CRAN (R 3.5.0)
##  compiler     3.5.1   2018-07-02 local
##  datasets   * 3.5.1   2018-07-02 local
##  devtools     1.13.6  2018-06-27 CRAN (R 3.5.1)
##  digest       0.6.15  2018-01-28 CRAN (R 3.5.0)
##  dplyr        0.7.6   2018-06-29 CRAN (R 3.5.1)
##  evaluate     0.10.1  2017-06-24 CRAN (R 3.5.0)
##  ggplot2    * 3.0.0   2018-07-03 CRAN (R 3.5.1)
##  glue         1.2.0   2017-10-29 CRAN (R 3.5.0)
##  graphics   * 3.5.1   2018-07-02 local
##  grDevices  * 3.5.1   2018-07-02 local
##  grid         3.5.1   2018-07-02 local
##  gtable       0.2.0   2016-02-26 CRAN (R 3.5.0)
##  htmltools    0.3.6   2017-04-28 CRAN (R 3.5.0)
##  knitr        1.20    2018-02-20 CRAN (R 3.5.0)
##  lazyeval     0.2.1   2017-10-29 CRAN (R 3.5.0)
##  magrittr     1.5     2014-11-22 CRAN (R 3.5.0)
##  memoise      1.1.0   2017-04-21 CRAN (R 3.5.1)
##  methods    * 3.5.1   2018-07-02 local
##  munsell      0.5.0   2018-06-12 CRAN (R 3.5.0)
##  pillar       1.2.3   2018-05-25 CRAN (R 3.5.0)
##  pkgconfig    2.0.1   2017-03-21 CRAN (R 3.5.0)
##  plyr         1.8.4   2016-06-08 CRAN (R 3.5.0)
##  purrr        0.2.5   2018-05-29 CRAN (R 3.5.0)
##  R6           2.2.2   2017-06-17 CRAN (R 3.5.0)
##  Rcpp         0.12.17 2018-05-18 CRAN (R 3.5.0)
##  rlang        0.2.1   2018-05-30 CRAN (R 3.5.0)
##  rmarkdown    1.10    2018-06-11 CRAN (R 3.5.1)
```

```
##  rprojroot     1.3-2   2018-01-03 CRAN (R 3.5.1)
##  scales        0.5.0   2017-08-24 CRAN (R 3.5.0)
##  stats       * 3.5.1   2018-07-02 local
##  stringi       1.2.3   2018-06-12 CRAN (R 3.5.0)
##  stringr       1.3.1   2018-05-10 CRAN (R 3.5.0)
##  tibble        1.4.2   2018-01-22 CRAN (R 3.5.0)
##  tidyselect    0.2.4   2018-02-26 CRAN (R 3.5.0)
##  tools         3.5.1   2018-07-02 local
##  utils       * 3.5.1   2018-07-02 local
##  withr         2.1.2   2018-03-15 CRAN (R 3.5.0)
##  yaml          2.2.0   2018-07-25 cran (@2.2.0)
```

# Loading and inspecting data

Let's start by reading data:

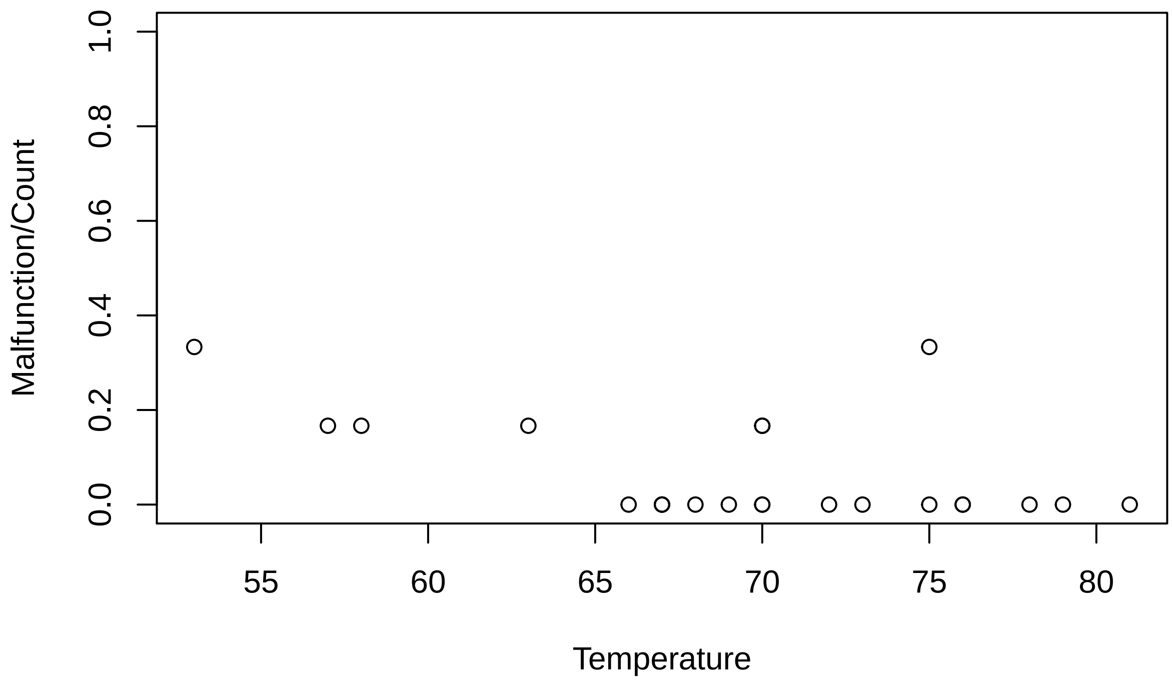
```
data = read.csv("../../data/shuttle.csv",header=T)
data
```

```
##          Date Count Temperature Pressure Malfunction
## 1     4/12/81     6          66       50           0
## 2    11/12/81     6          70       50           1
## 3     3/22/82     6          69       50           0
## 4    11/11/82     6          68       50           0
## 5     4/04/83     6          67       50           0
## 6     6/18/82     6          72       50           0
## 7     8/30/83     6          73      100           0
## 8    11/28/83     6          70      100           0
## 9     2/03/84     6          57      200           1
## 10    4/06/84     6          63      200           1
## 11    8/30/84     6          70      200           1
## 12   10/05/84     6          78      200           0
## 13   11/08/84     6          67      200           0
## 14    1/24/85     6          53      200           2
## 15    4/12/85     6          67      200           0
## 16    4/29/85     6          75      200           0
## 17    6/17/85     6          70      200           0
## 18 7/2903/85     6          81      200           0
## 19    8/27/85     6          76      200           0
## 20   10/03/85     6          79      200           0
## 21   10/30/85     6          75      200           2
## 22   11/26/85     6          76      200           0
## 23    1/12/86     6          58      200           1
```

We know from our previous experience on this data set that filtering data is a really bad idea. We will therefore process it as such.

Let's visually inspect how temperature affects malfunction:

```
plot(data=data, Malfunction/Count ~ Temperature, ylim=c(0,1))
```




## Logistic regression

Let's assume O-rings indpendently fail with the same probability which solely depends on temperature. A logistic regression should allow us to estimate the influence of temperature.

```
logistic_reg = glm(data=data, Malfunction/Count ~ Temperature, weights=Count,
                   family=binomial(link='logit'))
summary(logistic_reg)
```

```
##
## Call:
## glm(formula = Malfunction/Count ~ Temperature, family = binomial(link = "logit"),
##     data = data, weights = Count)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -0.95227  -0.78299  -0.54117  -0.04379   2.65152
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  5.08498    3.05247   1.666   0.0957 .
## Temperature -0.11560    0.04702  -2.458   0.0140 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 24.230  on 22  degrees of freedom
## Residual deviance: 18.086  on 21  degrees of freedom
## AIC: 35.647
##
```

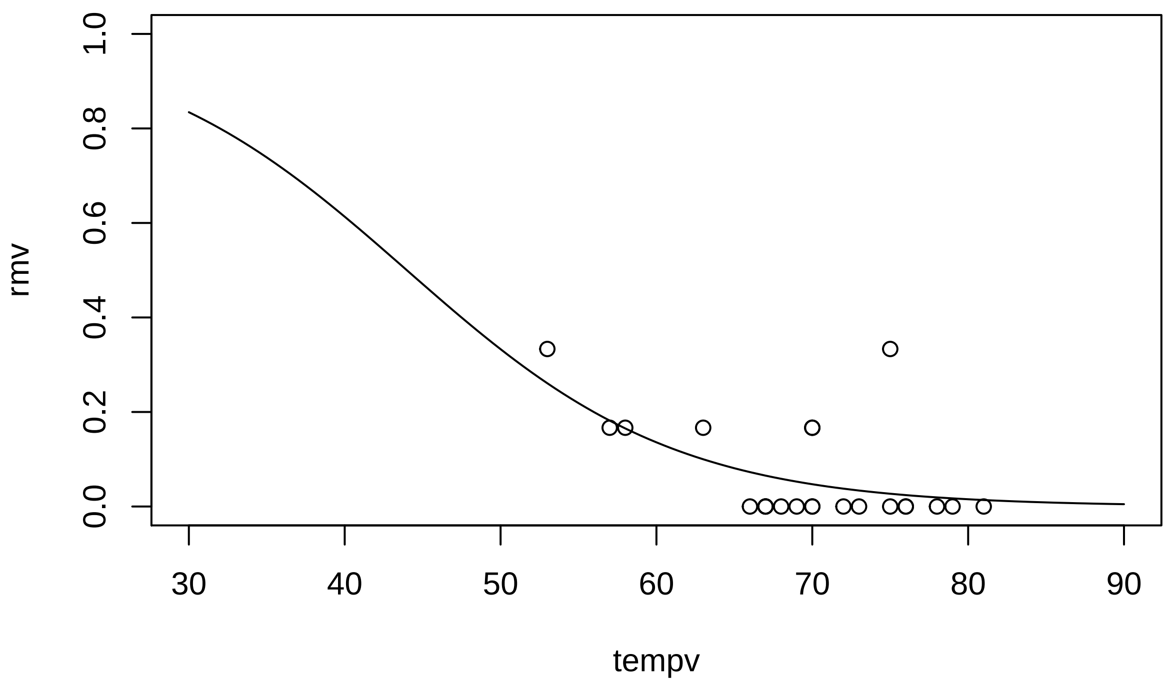
```
## Number of Fisher Scoring iterations: 5
```

The maximum likelyhood estimator of the intercept and of Temperature are thus $\hat{\alpha} = 5.0849$ and $\hat{\beta} = -0.1156$ and their standard errors are $s_{\hat{\alpha}} = 3.052$ and $s_{\hat{\beta}} = 0.04702$. The Residual deviance corresponds to the Goodness of fit $G^2 = 18.086$ with 21 degrees of freedom. **I have therefore managed to replicate the results of the Dalal et. al. article**.

# Predicting failure probability

The temperature when launching the shuttle was 31°F. Let's try to estimate the failure probability for such temperature using our model.:

```
# shuttle=shuttle[shuttle$r!=0,]
tempv = seq(from=30, to=90, by = .5)
rmv <- predict(logistic_reg,list(Temperature=tempv),type="response")
plot(tempv,rmv,type="l",ylim=c(0,1))
points(data=data, Malfunction/Count ~ Temperature)
```

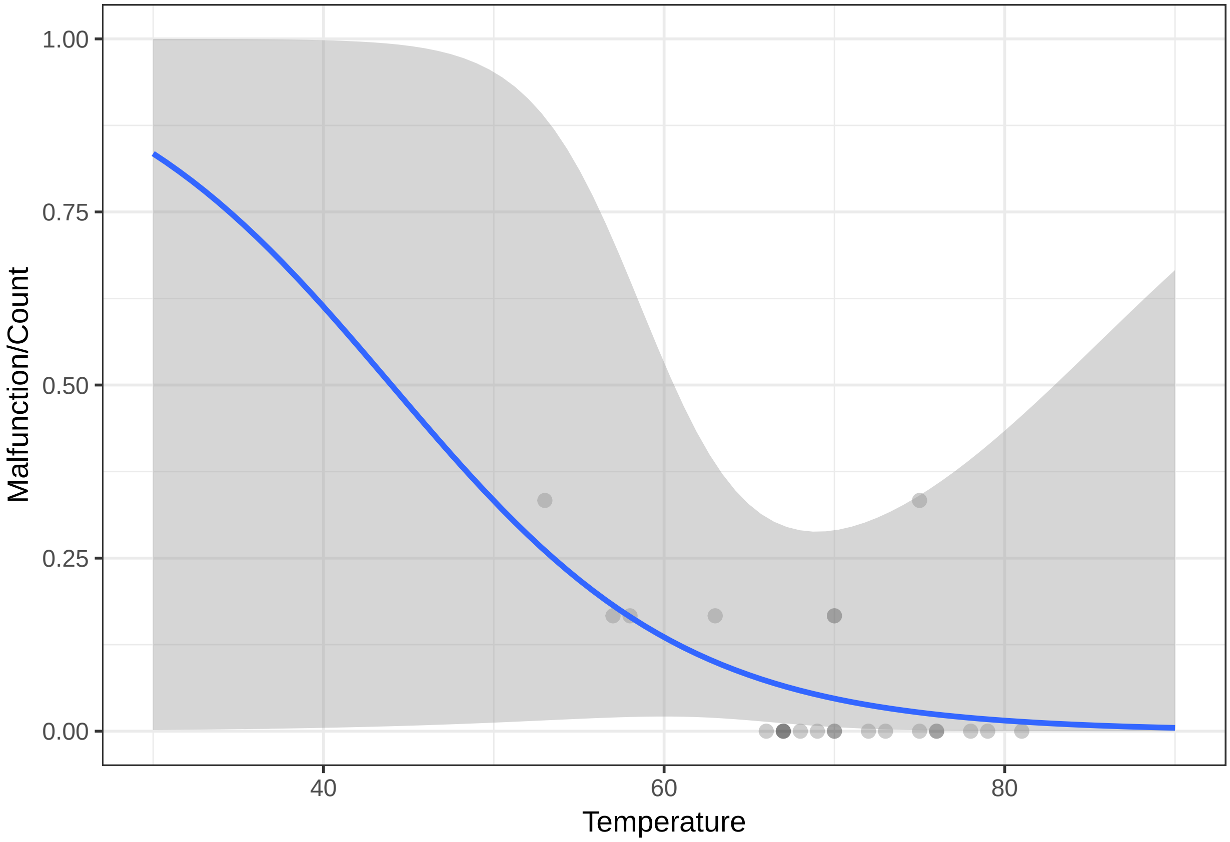



This figure is very similar to the Figure 4 of Dalal et al. **I have managed to replicate the Figure 4 of the Dalal et al. article.**

Let's try to plot confidence intervals although I am not sure exactly how they are computed.

```
ggplot(data, aes(y=Malfunction/Count, x=Temperature)) + geom_point(alpha=.2, size = 2) +
  geom_smooth(method = "glm", method.args = list(family = "binomial"), fullrange=T) +
  xlim(30,90) + ylim(0,1) + theme_bw()
```

```
## Warning in eval(family$initialize): non-integer #successes in a binomial
## glm!
```




No confidence region was given in the original article. **Let's hope this confidence region estimation is correct.**